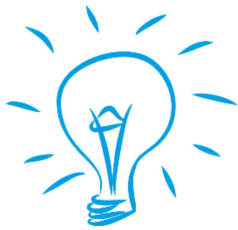




8. DEMAND FORECASTING



The chapter discusses forecasting theory. Particular attention was paid to demand forecasting. It is the prediction of future events (related to demand and demand), the aim of which is to minimize the risks associated with making business decisions. The most important issues discussed in this chapter include:

- forecasting principles and trends,
- time series forecasting,
- procedure for developing forecasts based on time series,
- forecasting methods and errors,
- the issue of artificial intelligence in forecasting.

8.1. Introduction

Forecasting is a widely used, multidisciplinary science. It is an important activity that is used to make business decisions in many areas of planning: economic, industrial and scientific (Chatfield, 2001). The built forecast supports making micro- and macroeconomic decisions. It also supports taking actions to activate or oppose some phenomenon. It is also a source of valuable information. Forecasting can be called prediction; predicting future demand, predicting sales or a new trend. Changes in market conditions to which the company must adapt can therefore be predicted.

However, this prediction cannot be based solely on the first and intuition of managers, which are predicted or not based on an improperly prepared basis, which can be triggered by the enterprise device. This is not every prediction is forecasting, because forecasting (also called prediction) is used on **rational, usually scientific grounds**.

Forecasting is inferring about unknown events based on known events (Cieślak, 2005). For example, it can be predicted that: (1) the event will occur because it occurred in the past;



(2) an event will occur because its frequency indicates it; (3) the event will occur because it is related to other events that have occurred (Dittmann, 2003).

Forecasts are developed (built) on the basis of premises of a very different nature. However, taking into account their scientific nature, they are constructed primarily on the basis of statistical and econometric models and using operational research. Forecasts are prepared using historical data - those that occurred in the past. And from the logistics point of view, they are related to data from the recent past. Especially in complex product industries (e.g. automotive industry), but not only, demand forecasts are crucial for the sales area and also for the efficiency of the production system.

The forecast always concerns a specific forecast horizon. The forecast horizon is the interval (T, T_i) , where: T – current moment, T_i – final moment.

Depending on the time horizon, the forecasting problem is generally divided into three areas: short-term, medium-term and long-term forecasting. As mentioned earlier, from the point of view of a logistics manager, **short-term forecasting** is crucial. It covers prediction horizons from one hour to a week. Also interesting from the point of view of a logistics manager's work is medium-term forecasting, which refers to forecasts from one month to a maximum of a year. Finally, we can distinguish long-term forecasts, which are characterized by a prediction horizon longer than one year. They are less important for operational activities related to logistics. Chaos theory has largely shown that long-term forecasting is a wasted effort. Therefore, it can be assumed that for broadly understood logistics activities, the longer the forecast horizon, the lower the probability of the forecast being made. Her confidence is decreasing. Also, forecasting for a product in a longer term than the product's life cycle does not make sense.

The value (importance) of forecasting models is based on their ability to produce accurate forecasts. Therefore, the forecasts are only as good as the assumptions of the model used. It is important to be aware and know what these assumptions are. If any of these assumptions turn out to be incorrect, the forecasts can be re-evaluated, modified and improved. The main problem of forecast accuracy is the unpredictability of economic trends



and external events and crises. Therefore, it should be clearly stated that specific forecast values are subject to **error** and **uncertainty**. Thus, the future is determined based on the knowledge we have about the past (Fig. 8.1).

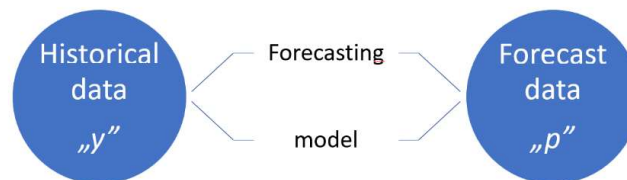


Figure 8.1. Generalized forecasting model

Source: (Dittmann, 2003)

We should not forget about the need to exchange information in supply chain management, which is crucial for the success of demand forecasting (Altendorfer & Felberbauer, 2023). The more accurate the information about demand, the more accurate the forecast will be. Also crucial is the ongoing update of information about demand (it involves changing previous information, e.g. about the size of the order), thanks to which demand is updated in the time horizon and the elimination of information asymmetry. Ali et al. point out that sharing full demand forecasts, rather than the final order quantity, is beneficial to supply chain performance (Ali et al., 2012).

The challenges of successful forecasting are more than just the technical difficulties of developing an accurate forecast model. Forecast models must be developed with a clear understanding of both the nature of the situation for which a forecast is desired and the resources available to produce the forecast. It is important to ensure that the selected variable relates directly to the forecast data needed (Sheldon, 1993). This does not mean that forecasts are useless, but that those who use them should constantly monitor their operating environment to detect any factors that indicate inconsistent or irregular patterns.

Although the forecast is subject to inaccuracies, it constitutes an important guideline for the future operational activities of the company. The justification for creating forecasts in an enterprise is also the cyclicity that occurs in enterprise operations. We predict that if



an event occurred in the past, it may also occur in the future. However, if an event occurred in the past with a certain frequency, the probability that it will occur again increases. Despite many uncertainties, a forecast constructed using scientific methods is a prerequisite for making a rational decision regarding the company's operations.

Economic practice also shows that a **simple forecasting method does not automatically mean a worse method** (Kucharski, 2013). As Kucharski points out, naive methods can forecast the same data with similar accuracy. They are much easier to use. As a result of activities related to demand forecasting, it is possible for the organization to obtain many benefits (Table 8.1).

Table 8.1. Benefits of demand forecasting

Identified forecasting benefit	Justification
Better production organization	knowing the forecast sales volume of finished products, the organization can plan the appropriate production volume and the appropriate demand for raw materials and packaging in advance; thus eliminating shortages on the production line
Greater control of safety stock	knowing the forecast sales volume of finished products, you can plan a safety stock that will guarantee the coverage of market demand
More effective reduction of outdated assortment	knowing the forecast sales volume of finished products, you can focus on servicing only the assortment necessary to cover demand; obsolete products can be eliminated and, as a result, the costs of frozen capital in inventory and storage costs can be optimized
Better customer satisfaction and improvement of the organization's image	knowing the forecast sales volume of finished products can ensure that the appropriate level of inventory is maintained in the warehouse
More efficient use of warehouse space	knowing the forecast sales volume of finished products, you can collect only the necessary stocks of products; you can also significantly reduce the storage space used
More effective control and cost minimization	knowing the forecast sales volume of finished products, you can more accurately plan the organization's budget and take steps to control expenses more precisely

Source: (Wojciechowski & Wojciechowska, 2015; Wolny & Kmiecik, 2020)

Several properties of forecasts should be indicated. These are:



1. Forecasts are formulated using the achievements of science (developed and verified mathematical models).
2. Forecasts refer to a specific future.
3. Forecasts are verified empirically (after a specified period of time).
4. Forecasts are acceptable to the person preparing the forecast.

Forecasts support the decision-making process in the company and at the same time fulfil various functions. (Gajda, 2001):

- preparatory – the forecast is an impulse to take a specific action, but it has no influence on the forecasted phenomenon. Only economic decisions are made on its basis,
- activating – the forecast is an impulse to take a specific action and at the same time influences the forecasted phenomenon. Therefore, actions are taken that are aimed at making the forecast realistic (self-fulfilling or favourable forecasts, which trigger actions that favour the realization of the forecasts) or annihilating the forecasts (warning forecasts, which trigger actions that counteract their realization).

However, it is important to remember that the forecasts built can easily break down due to random variables that cannot be incorporated into the model or may be simply wrong from the start. For this reason, forecasting can be dangerous for organizations. There are three problems related to forecasting:

- the data on the basis of which forecasts are made will always be old, relating to historical periods. So there is never a guarantee that past conditions will persist in the future,
- exceptional or unexpected events or external effects cannot be taken into account (example of the COVID-19 pandemic; impact of war and armed conflicts; impact of unforeseen economic crises),
- forecasts cannot take into account their own impact.



Properly conducted forecasting allows entrepreneurs and managers to plan their activities in advance, increasing the chances of remaining competitive on the markets.

8.2. Classification of forecasting methods

There are two basic groups of forecasting methods: quantitative and qualitative (Fig. 8.2). A forecast classified as a quantitative forecasting method takes the form of a specific number (point forecast) or, alternatively, a numerical range (interval forecast).

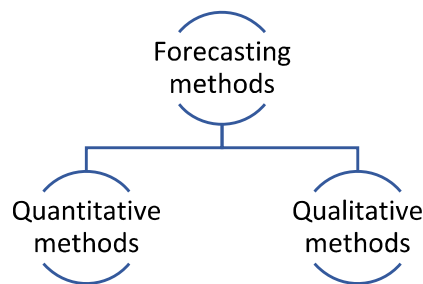


Figure 8.2. Forecasting methods – types

Source: (Dittmann, 2000)

Qualitative forecasts take a non-numerical form. They refer to the analysed phenomenon in the future and the estimation of its growth, decline or no change. Qualitative forecasts can be treated as based on the opinions of market experts.

From the point of view of a logistics specialist, however, the key forecasts are those that can be defined using numbers, i.e. **quantitative forecasts**. Quantitative forecasting bypasses the expert factor and tries to remove the human element from the analysis. These approaches focus solely on data.



Quantitative forecasts	Time series models
	Econometric models
	Analog models
	Lead variable models
	Cohort analysis models
	Market tests

Figure 8.3. Quantitative forecast methods

Source: (Dittmann, 2000)

Quantitative forecasts can be classified according to the models used (Fig. 8.3). For the purposes of this book, the focus is on **time series models**.

8.3. Time series forecasting

One of the most frequently used forecasting methods for forecasting demand are methods based on time series models. Time series is a methodology for exploring complex and sequential types of data. In time series models, sequential data, which consists of strings of numeric data, is recorded at regular intervals (e.g. per minute, per hour, or per day). The popularity of these methods results from the possibility of obtaining information about the future course of the observed phenomenon through forecasting. Therefore, there is no need to collect and analyse further data from other sources. Forecasting using time series is also often used due to the high probability of its occurrence. Economic practice also shows that forecasts prepared using time series models are not worse than forecasts obtained based on more complicated models. Experience also shows that **time series models have development potential**. Each subsequent modification of the method or subsequent time series forecasting method should, by definition, improve the quality of its results.

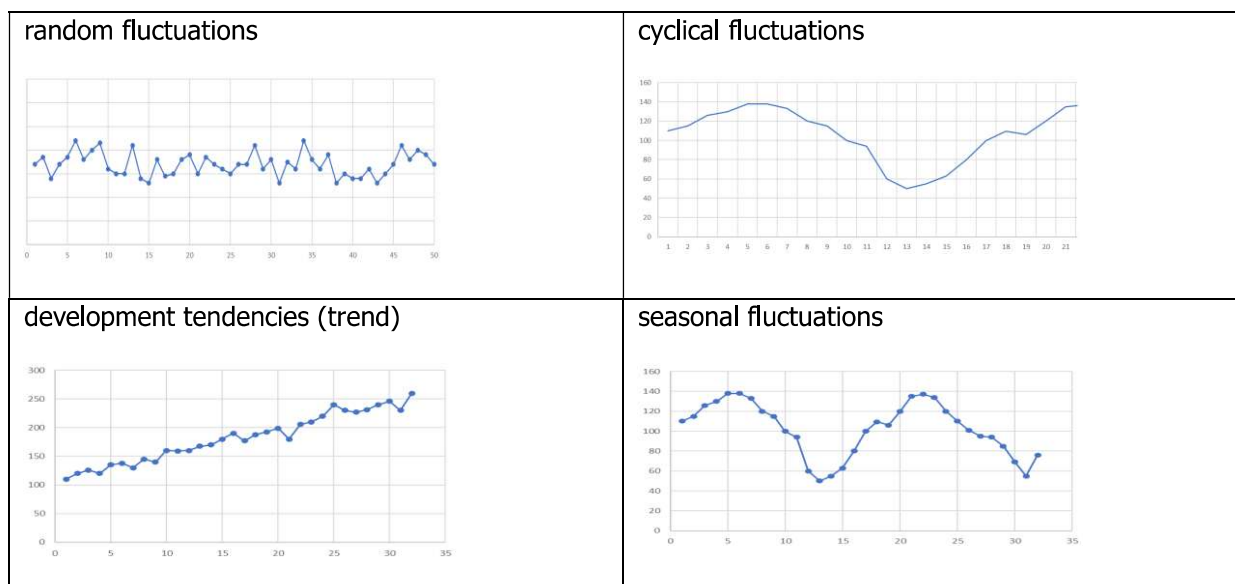


8.4. Time series decomposition

Forecasts are built using time series data. This happens regardless of the forecasting method adopted. Time series data (variables) are ordered chronologically, from the oldest data to the newest data. It should be emphasized that the last data does not correspond to the moment of building the forecast. In scientific publications and studies, it is assumed that y_t always determines a specific value of the y series in the period (moment) t .

The components of the time series are random fluctuations, development tendency (trend), cyclical fluctuations and seasonal fluctuations (Table 8.2).

Table 8.2. Time series visualization



Source: own study

A few words can be said about each of them (Cieślak, 1997):

- random fluctuations – these are random, accidental and unpredictable changes in a series variable of varying strength, which are observed over time and do not show any clear tendency. They are associated with errors of a statistical or prognostic nature,



- development tendencies (trend) – these are the long-term tendency of series data to one-way (monotonic) changes in the forecast variable. They are of an increase or decrease nature. They most often concern a permanent phenomenon that affects the analysed data. Time series data may contain both developmental trends and random fluctuations. In order to isolate development trends, more historical data is usually needed. Hence, we observe a rule of thumb: the longer the period of historical data observation, the greater the ability to precisely determine the type of trend. The trend is presented using a linear or non-linear mathematical function,
- cyclical fluctuations – these are long-term, rhythmic fluctuations in the value of a variable around a trend or a constant level, which persist for a long time (longer than a year). They are the result of business cycles. Different cycle lengths and their dynamics can be observed. Cyclical fluctuations are therefore related to changes in the economic activity of enterprises, crises or economic recovery or the wealth of society. To analyse cyclical fluctuations and build a forecast of future demand, monthly, quarterly or annual historical data from the last few years are needed,
- seasonal fluctuations – these are fluctuations in the value of a time series variable around a trend or constant level, which tend to repeat at regular (seasonal) intervals, not exceeding a year. In such a case, the accuracy of the forecasts built will depend on the type and scale of seasonal fluctuations, the number and type of gaps in the available data, and the forecast horizon.

The identification and analysis of the indicated components of a time series is called **time series decomposition**.

8.5. Preparing time series data

Before taking steps to build forecasts, it is worth performing preliminary data processing, also known as **data cleaning**. They need to be verified to eliminate errors or outliers. Skipping this step may result in distortion of forecast results and, as a result, errors in conclusions.



It must be remembered that data related to unusual cases (outliers or rare cases) are true information about which the forecaster has no doubts. They are proven and reliable.

There are various strategies for dealing with outlier data. This is among other things:

- no action – it involves ignoring atypical data, because some forecasting methods are resistant to the occurrence of atypical data,
- filtering cases with outlier data – this involves removing this data; however, this is not the best strategy,
- replacing unusual data – this is a popular strategy in which outliers are replaced by: (1) a value of 0, (2) an average value; (3) the maximum/minimum value of the filter or (4) another value determined on the basis of a substantive criterion.

Table 8.3. Selected types of outliers

Additive outlier It appears as a surprisingly large or surprisingly small value for a single observation. It has no effect on subsequent observations – the value of the series does not deviate any further.	
Innovative outlier It occurs as a deviation with further effects on observations. An initial (first) effect with a delay and extension effect on subsequent observations (decreasing or increasing) can be observed. This impact may decrease or increase over time.	
Transient change outlier It occurs when the influence decays exponentially with subsequent observations. Eventually the series returns to its normal level.	
Seasonally additive outlier It appears as a surprisingly large or surprisingly small value that occurs periodically (at regular intervals).	

Source: own study



Leaving atypical data in the time series distorts the result of their analysis and makes it difficult to formulate conclusions, because the atypical data are extremely small or extremely large values. Due to their inconsistency, these values are called **outliers**. As a result, they increase the range in the time series (minimum-maximum range). Therefore, atypical data have a large distorting effect on the forecast value (Table 8.3).

The decision to change the size of an unusual item or remove it is always highly subjective to the forecaster, so it requires caution. Reducing the forecaster's subjectivity when removing atypical cases is possible with quantitative data. For example, you can apply the **standard deviation rule**. This means that if historical data is unusual (e.g. outside the range of the group mean ($\bar{x}$) plus or minus 2 or 3 standard deviations), it is changed or removed.

It is worth submitting each series of data to the decomposition procedure. Several steps can be specified:

1. Identifying the functional form of the series, which means determining the type of trend.
2. Search for outlier observations and replace them with average values or so-called upper and/or lower filters.
3. Verification whether the last observed data in the series behaves typically; if not cleansing it.
4. Identification of the trend slope coefficient, in order to determine the stability of the main trend observed in the series.
5. Examine the stability of the current short-term trend.

Filters (minimum or maximum) are a good solution when outliers appear (Fig. 8.4).

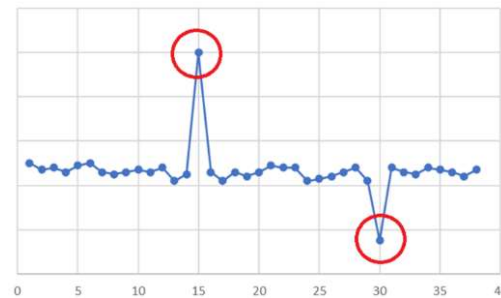


Figure 8.4. Clearly inconsistent values with the general regularity of the time series

Source: own study

The filter is used to correct the data before building the next forecast. A value that deviates from the correct series is replaced with a minimum or maximum filter (Fig. 8.5). Extreme outliers are found above $y_{max} + 3R_q$ or below $y_{min} - 3R_q$. Values suspected of being outliers are included in the ranges $(y_{min} - 1,5R_q; y_{min} - 3R_q)$ and $(y_{max} + 1,5R_q; y_{max} + 3R_q)$.

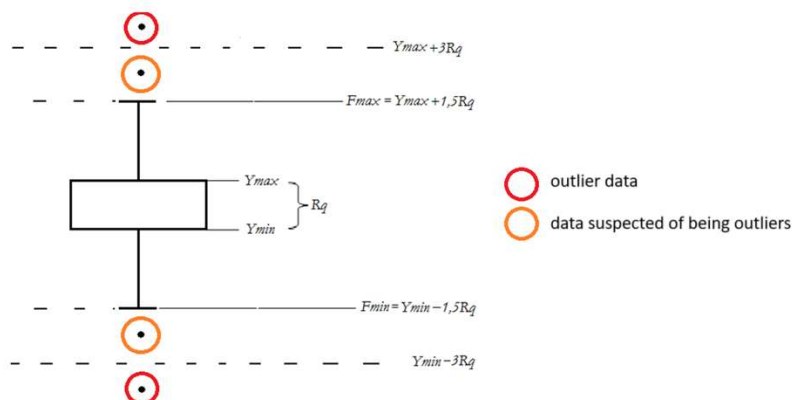


Figure 8.5. Filter values and outliers

Source: (Grzybowska, 2009)

This is represented by the model:

$$F_{min} = y_{min} - 1,5R_q$$

$$F_{max} = y_{max} + 1,5R_q$$

$$R_q = y_{max} - y_{min}$$



where:

F_{min} – minimum filter value,

F_{max} – maximum filter value,

y_{min} – minimum value determined from the time series,

y_{max} – maximum value determined from the time series,

R_q – interquartile range.

8.6. Time series forecasting methods

Time series forecasting methods have been divided according to the data trend. Forecasts can be specified for constant demand, trend-like demand (growing or decreasing) and seasonal demand (Fig. 8.6).

Constant demand	Naive methods
	Average methods
	Exponential smoothing method (Brown's model)
	Model ARMA
Trend-like demand	Linear exponential smoothing. LES (Holt's model)
	Simple linear regression method
	Model ARIMA
	Model RW
Seasonal demand	Holt-Winters' seasonal method (Holt-Winters' model)

Figure 8.6. Quantitative methods for time series forecasting

Source: own study

In the presented models, the y parameter always refers to the actual demand values, and the $\hat{y}_t$ parameter always refers to the constructed forecast.



Naive methods

Naive forecasting methods are characterized as simple, fast and cheap. They allow the development of forecasts from a small amount of historical data. Naive methods also serve as a reference point for other forecasting methods (Kucharski, 2013).

Naive methods are the simplest mechanical methods. They have been developed on the assumption that there will be no significant changes in demand in the future. They are very suitable wherever there are no large fluctuations in the forecast variable. They are based solely on historical observations. Naive models only have memory of one (last) observation, so they will not filter out noise in the data, but rather copy it into the future.

Naive models consist of simple projective models. This means that they require input from recent observations and no statistical analysis is performed. They are extremely simple and at the same time surprisingly effective. The advantage of these methods is a quick decision about the predicted value. However, the disadvantage is the inability to analyse the cause and effect relationships that underlie the forecast variable.

For the purposes of this study, 3 naive methods will be presented: (1) Naïve Forecast; (2) Seasonal Naïve Method; (3) Drift Method.

Naive forecast

A naive forecast is one in which the forecast value for a given period is simply equal to the value observed in the previous period. This is represented by the formal model:

$$\check{y}_t = y_{t-1}$$

where:

$\check{y}_t$ – forecast for the future period,

y_{t-1} – actual value of demand from the previous period.



Formula used in Excel:

$$\text{forecast}_{(t)} = \text{demand}_{(t-1)}$$

Seasonal Naive method

Also, the naive method is useful for data with small seasonal fluctuations. In this situation, each forecast is equal to the last observed value from the same season (e.g. from the same month of the previous year). The forecasts assume the value observed in the previous season. The model is useful when there are small random fluctuations and additive seasonality. This is represented by the formal model:

$$\check{y}_{t+h|T} = y_{t+h-m(k+1)}$$

where:

$\check{y}_{t+h|T}$ – forecast for the future period,

m – seasonal period,

k – total part $(h-1)/m$ (i.e. the number of complete years in the forecast period preceding $T+h$).

The model looks more complicated than it actually is. For example, if a forecast is built based on monthly data, it applies to all future monthly values and is equal to the last observed value for that month of the previous year. For quarterly data, the forecast of all future $Q2$ values is equal to the last observed $Q2$ value (where $Q2$ represents the second quarter). Similar rules apply in other months and quarters and in other seasonal periods.



This is explained by the formula used in Excel:

$$\text{forecast}_{(t)} = \text{demand}_{(t, \text{previous year})}$$



The forecast for period t is equal to the demand from the corresponding period of the previous year. The seasonal naive method will require a one-year lag.

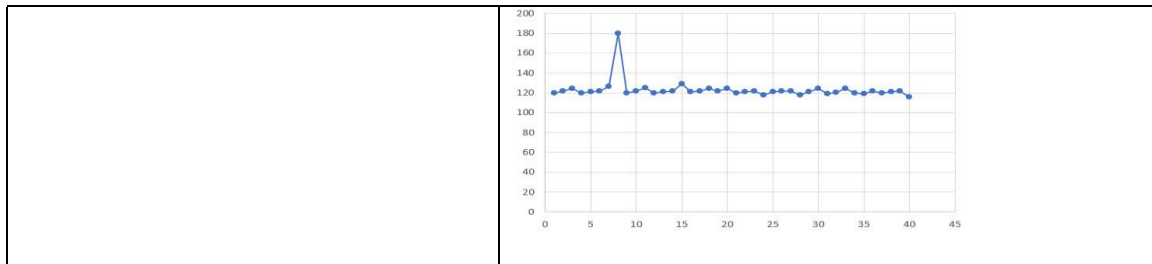
Drift method

A variation of the seasonal naive method, the drift method involves allowing forecasts to increase or decrease over time, where the amount of change over time (called drift) is set to the average change seen in historical data. The method uses an additional component called drift. **Drift** refers to the decline in model performance due to changes in data and the relationship between input and output variables. This, however, may result in deterioration of the quality of the forecasting model, resulting in inaccurate forecasts. Variable drift refers to changes in input values, e.g. resulting from sudden changes in sales trends. Changing the input data can be:

- violent (sudden), e.g. due to lockdowns during the COVID-19 pandemic,
- increasing (changing slowly),
- impulse (one-off), e.g. in the case of incorrectly supplied data.

Table 8.4. Types of drift

Change of input data – rapid (sudden)	
Change of input data – incremental	
Change of input data – impulse	



Source: own study

This is represented by the formal model:

$$\tilde{y}_{t+h|T} = y_t + h \left(\frac{y_t - y_1}{t - 1} \right)$$

where:

$\tilde{y}_{t+h|T}$ – forecast for the future period,

$h \left(\frac{y_t - y_1}{t - 1} \right)$ – drift component.

This is equivalent to drawing a line between the first and last observation and extrapolating it into the future.

This is explained by the formula used in Excel:

forecast_(t) = demand_(from the last period of the previous year) + [h * (demand_(from the last period of the previous year) – demand_(from the first period of the previous year)] / n - 1



The forecast for period t is equal to the demand from the last period of the previous year + the drift component. The drift component contains the number h , which refers to the next number of forecasts built, and t determines the number of examined periods in the year.



Average methods

Moving average methods act as a filter because they eliminate short-term fluctuations from the data series. To build a forecast, moving average methods use a specific number of adjacent demand data.

As the number of historical data on the basis of which the forecast is built increases, the smoothing effect increases. This means that using more data in the model smooths the series more strongly. It also causes a slower reaction to changes in the level of the forecast variable. And conversely. Using less historical data helps reflect changes in demand from recent periods more quickly. The forecast then becomes more sensitive to random fluctuations for the purposes of this study, 4 average methods will be presented: (1) Global Mean or Global Average; (2) Simple Moving Average, SMA; (3) Exponential moving average, EMA; (4) Weighted Moving Average, WMA.

Global Mean or Global Average

The forecast, using the global average method, is built on all available historical observations included in the series. This method determines the central tendency, which is the location of the centre of the data series in the statistical distribution.

The forecast value using the global average method should be calculated by adding the values of all historical data and dividing the calculated value by the number of analysed periods.

This is represented by the formal model:

$$\tilde{y}_t = \frac{\sum_{t=1}^n y_t}{n}$$

where:

n – number of periods analysed.



This is explained by the formula used in Excel:

$$\text{forecast}_{(t)} = \Sigma \text{demand} / n$$

Using the arithmetic mean of the entire series of data distorts the result of the forecast being built. The data used in the method is too outdated, which distorts the correct picture of future demand.

Simple Moving Average, SMA

The simple moving average method uses a typical arithmetic average, but only of a certain amount of historical data. Curated data helps smooth out demand data, reducing the impact of random fluctuations and outdated data. The name moving average means that each forecast is calculated based on data from the previous x periods. A simple moving average does not distinguish historical data and does not weight that data.

A simple moving average is an arithmetic moving average calculated by adding the latest demand data and then dividing the resulting value by the number of periods in the calculated average. This is represented by the formal model:

$$\tilde{y}_t = \frac{\sum_{t+1-m}^t y}{m}$$

where:

m – number of periods analysed.

This is explained by the formula used in Excel:

for a 3-element average:



$$\text{forecast}_{(t)} = \text{AVERAGE}(\text{demand}_{(t-1)}; \text{demand}_{(t-2)}; \text{demand}_{(t-3)};)$$

To calculate a moving average you can use a simple formula based on the *AVERAGE* function with relative references.



As the formulas are copied down the column, the range changes on each row to account for the values needed for each average.

The longer the moving average, the greater the **lag**. This can be explained as follows:

- The forecast based on 3 historical data is a short-term moving average; it is like a motorboat – agile and quickly changing.
- The forecast based on 50 historical data is a long-term moving average; it is like an ocean tanker – sluggish and slow to change.
- Therefore, the lag factor should be kept in mind when selecting the appropriate number of historical periods (do not use too much data).

Exponential Moving Average, EMA

The exponential moving average method allows you to reduce latency by paying more attention to recent historical data values. This makes the method more responsive to recent data values. An exponential moving average is usually more sensitive to recent changes in demand compared to a simple moving average. This is represented by the formal model:

$$\tilde{y}_t = p_{t-1} + \alpha(y_{t-1} - p_{t-1})$$

$$\alpha = \frac{2}{n + 1}$$

where:

α – multiplier,

n – selected time period.

Calculating an exponential moving average involves three steps:

1. Calculation of a simple moving average for a period (preliminary forecast). The SMA is only required to provide an initial value for further calculations.
2. Calculating a multiplier for weighting an exponential moving average.



Example: if the forecast is built from 3 periods, the multiplier will be calculated

as follows: multiplier = $\alpha = \frac{2}{n+1} = \frac{2}{3+1} = 0,5$

3. Calculation of the current forecast according to an exponential moving average

Reminder: The first forecast calculated is called the preliminary forecast.



This is explained by the formula used in Excel:

forecast_(t) = forecast_(t-1) + multiplier * (demand_(t-1) – forecast_(t-1))

The EMA method is used to capture shorter trend moves due to its focus on the latest data and recent forecasts.

Weighted Moving Average, WMA

This is a variant of the Simple Moving Average (SMA). The Weighted Moving Average (WMA) method is a moving average that gives weight to the latest demand values. This means that the latest data has the strongest impact on the forecast value than older data. This is possible by using a weighted factor. The use of weighted coefficients allows for more accurate forecasts. The method is considered more sensitive to changes in demand.

Weighted moving average forecasts are obtained by multiplying each demand value by a predetermined weighted factor and summing the resulting values. This is represented by the formal model:

$$\tilde{y}_t = \sum_{i=t+1-m}^t (y_i \cdot \varpi_i)$$

where:

ϖ – weighted coefficient.

To determine their value, remember a few rules:



- The values of the weighted coefficients are in the range $< 0,1 >$,
- Each subsequent weighted coefficient used is greater than its predecessor $\varpi_i < \varpi_{i+1} < \varpi_{i+2}$. This is a very important principle because it differentiates the importance of the historical data used. Older ones have a lower weighted factor, newer data are more important and have a higher weighted factor,
- The sum of all weighted coefficients must equal 1: $\sum_1^n \varpi_i = 1$,
- The number of weighted coefficients depends on the number of analysed historical periods from the time series.

This is explained by the formula used in Excel

for a 3-element weighted average:



$$\text{forecast}_{(t)} = (\text{demand}_{(t-3)} * \varpi_{(1)}) + (\text{demand}_{(t-2)} * \varpi_{(2)}) + (\text{demand}_{(t-1)} * \varpi_{(3)})$$

for a 5-element weighted average:

$$\text{forecast}_{(t)} = (\text{demand}_{(t-5)} * \varpi_{(1)}) + (\text{demand}_{(t-4)} * \varpi_{(2)}) + (\text{demand}_{(t-3)} * \varpi_{(3)}) + (\text{demand}_{(t-2)} * \varpi_{(4)}) + (\text{demand}_{(t-1)} * \varpi_{(5)})$$

In the weighted moving average method, the value of the smoothing constant should be determined (how many historical periods should be used) and the levels of individual weights of the weighted coefficients should be determined.

Exponential Smoothing methods

Exponential smoothing methods are widely used (Chatfield et al., 2001). There are 15 different methods. Each variant is specified for a different forecasting scenario. The most well-known variants of the exponential smoothing method are Simple Exponential Smoothing (SES) (no trend, no seasonality), Holt's linear method (additive trend, no seasonality),



Holt-Winters additive method (additive trend, additive seasonality) and Holt-Winters multiplicative method (additive trend, seasonality). multiplicative) (De Gooijer & Hyndman, 2006). Researchers have proposed numerous variants of the original exponential smoothing methods, e.g., Carreno and Madinaveitia (1990) proposed modifications to deal with discontinuities, and Rosas and Guerrero (1994) looked at exponential smoothing predictions subject to one or more constraints. For the purposes of this study, three exponential smoothing methods will be presented: (1) Simple Exponential Smoothing, SES (model Browna); (2) Linear Exponential Smoothing, LES (model Holta); (3) Holt-Winters' seasonal method (Model Holt-Winters).

Simple Exponential Smoothing, SES (Brown's model)

Simple exponential smoothing is the basic form of exponential smoothing. The exponential smoothing method (Brown's model) is a relatively accurate method of forecasting demand. It takes into account the exponential smoothing factor (α). This coefficient controls the rate at which data influences the forecasts being built. At the same time, the method gives more weight to newer data. It assigns exponentially decreasing weights as the data becomes more distant.

To determine the value of the exponential smoothing factor, remember the following rules:

- the value of the exponential smoothing coefficient is in the range of $(0,1)$;
- the value of the exponential smoothing coefficient is selected experimentally. The following assumption should be used: the closer the coefficient is to 0, the more the data is smoothed (the forecast is less sensitive to changes in demand), and the more the forecaster trusts the forecast made in the previous period; an indicator closer to 1 means that the forecast is more sensitive to changes in demand, and the forecaster is based on the actual situation that occurred in the previous period.



This is represented by the formal model:

$$\tilde{y}_t = a \cdot y_{t-1} + (1 - a) \cdot p_{t-1}$$

where:

a – exponential smoothing factor.

Calculating the forecast using the simple exponential smoothing method involves three steps:

1. Calculation of the naive forecast for the first period (initial forecast). The preliminary forecast is only required to provide an initial value for further calculations.
2. Determining the exponential smoothing factor; Exponential smoothing factor $a = \langle 0,1 \rangle$
3. Calculation of the forecast according to the simple exponential smoothing method.



Formula used in Excel:

$$\text{forecast}_{(t)} = \text{alfa} * \text{demand}_{(t-1)} + [(1 - \text{alfa}) * \text{forecast}_{(t-1)}]$$

The simple exponential smoothing method is used to forecast based on data without any significant trend or seasonality.

Linear Exponential Smoothing, LES (Holt's model)

The double exponential smoothing method (the so-called Holt model) captures linear trends in the data. It is the right model for demand in which a constant upward or downward trend can be observed. However, there is no seasonality.

The double exponential smoothing model is based on two smoothing factors. One of them refers to the smoothing of the variable level (random fluctuations), and the other to its increase (trend fluctuations). Both coefficients should be within the range: $\langle 0,1 \rangle$. This is represented by the formal model:

$$\tilde{y}_t = L_{t-1} + T_{t-1}$$



$$L_t = \alpha y_{t-1} + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_{t-1} - L_{t-2}) + (1 - \beta)T_{t-1}$$

where:

α – variable level smoothing factor,

β – growth smoothing factor.

According to the model, two starting values L_t i T_t are needed:

$L_t = y_{t-1}$ – according to the naive method

$T_t = y_{t-1} - y_{t-2}$

Formula used in Excel:



forecast_(t) = [alfa * demand_(t-1) + (1 – alfa)(random fluctuation_(t-1) + trend fluctuation_(t-1))] + [beta * (random fluctuation_(t-1) – (random fluctuation_(t-2)) + (1 – beta) * trend fluctuation_(t-1)]

Holt-Winters' Seasonal method (Holt-Winters' Model)

The seasonal exponential smoothing method, also called the Holt-Winters model, is very suitable for forecasting demand for data characterized by both trend and seasonality (www_8.1). However, it is necessary to obtain long series of demand data because it is necessary to verify repeated cyclical fluctuations (confirming that there is seasonal demand in the series). Seasonal fluctuations occur in additive or multiplicative versions (Kucharski, 2013).

Additive fluctuations occur when, in individual sub-periods of the seasonality cycle, deviations of the level of the analysed phenomenon from the average level or trend, in terms of absolute value, can be observed.

This is represented by the formal model:

$$\check{y}_t = L_{t-1} + T_{t-1} + S_{t-p}$$

$$L_t = \alpha(y_t - S_{t-p}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$



$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \delta(y_t - L_t) + (1 - \delta)S_{t-p}$$

where:

α – variable level smoothing factor,

β – trend smoothing factor,

δ – coefficient of the seasonal component,

L_t – level component at time t ,

T_t – trend component at time t ,

S_t – seasonal component at time t ,

p – seasonal period.

Formula used in Excel:

**forecast_(t) = level component_(t-1) + trend component_(t-1) +
seasonality component_(t-p)**

**level component_(t) = alpha * [demand_(t) – seasonality component_(t)]
+ (1 – alpha) * (level component_(t-1) + trend component_(t-1))**

trend component_(t) = beta * (level component_(t) - level component_(t-1)) + [(1 – beta) * trend component_(t-1)]

seasonality component_(t) = gamma * (demand_(t) – level component_(t)) + (1 – gamma) * seasonality component_(t-p)

seasonality component_(t-p) for four quarters

seasonality component₍₁₎ = demand₍₁₎ / average demand₍₁₋₄₎

seasonality component₍₂₎ = demand₍₂₎ / average demand₍₁₋₄₎

seasonality component₍₃₎ = demand₍₃₎ / average demand₍₁₋₄₎

seasonality component₍₄₎ = demand₍₄₎ / average demand₍₁₋₄₎



Multiplicative fluctuations occur when, in individual sub-periods of the cycle, a deviation from the average level or trend by a certain constant relative amount can be observed (Sobczyk, 2006). This is represented by the formal model:

$$\check{y}_t = (L_{t-1} + T_{t-1})S_{t-p}$$

$$L_t = \alpha \left(\frac{y_t}{S_{t-p}} \right) + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \delta \left(\frac{y_t}{L_t} \right) + (1 - \delta)S_{t-p}$$

where:

α – variable level smoothing factor,

β – trend smoothing factor,

δ – coefficient of the seasonal component,

L_t – level component at time t ,

T_t – trend component at time t ,

S_t – seasonal component at time t ,

p – seasonal period.

It is worth remembering that: an additive trend is related to double exponential smoothing with a linear trend, while a multiplicative trend is related to double exponential smoothing with an exponential trend (www_8.2).



Formula used in Excel:

**forecast_(t) = (level component_(t-1) + trend component_(t-1)) *
seasonality component_(t-p)**



level component_(t) = alpha * [demand_(t) / seasonality component_(t)] + (1 – alpha) * (level component_(t-1) + trend component_(t-1))

trend component_(t) = beta * (level component_(t) – level component_(t-1)) + [(1 – beta) * trend component_(t-1)]

seasonality component_(t) = gamma * (demand_(t) / level component_(t)) + (1 – gamma) * seasonality component_(t-p)

seasonality component_(t-p) for four quarters

seasonality component₍₁₎ = demand₍₁₎ / average demand₍₁₋₄₎

seasonality component₍₂₎ = demand₍₂₎ / average demand₍₁₋₄₎

seasonality component₍₃₎ = demand₍₃₎ / average demand₍₁₋₄₎

seasonality component₍₄₎ = demand₍₄₎ / average demand₍₁₋₄₎

Autoregressive methods

Among the forecasting models, models of stationary ARMA series (AutoRegressive Moving Average) and non-stationary ARIMA series (AutoRegressive Integrated Moving Average model) have a special place. These are models based on the phenomenon of autocorrelation, created by integrating the AR autoregressive model (AutoRegressive model) and the MA (Moving Average) model (Grzelak, 2019).

For the purposes of this study, 3 autoregressive methods will be presented: (1) Autoregressive Moving Average (ARMA); (2) Autoregressive Integrated Moving Average (ARIMA); (3) Random Walk (RW).

The ARMA and ARIMA models have many similarities. The components AR(p) – general autoregressive model and MA – general moving average model MA(q) are the same. What distinguishes both ARMA and ARIMA models is the difference. If there are no differences in an ARMA model, it simply becomes an ARIMA model.



Autoregressive Moving Average, ARMA

The requirement for forecasting according to the ARMA method is a series of data that is characterized by stationarity. It means that in this series we can distinguish a constant mean, a constant variance and a constant covariance, which depends only on the time interval between the values (Schaffer et al., 2021).

According to the ARMA model, the forecast value at time t depends on its past values and on the differences between the past actual values of the forecast variable and its values obtained from the model – forecast errors. This model is referred to as the **ARMA(p,q)** model, where p refers to the order of the autoregressive polynomial and q is the order of the moving average polynomial. This is represented by the formal model:

$$\tilde{y}_t = \varepsilon_t + (\alpha y_{t-1} + \varepsilon_t) + (\beta y_{t-2} - \alpha y_{t-1} + \varepsilon_t) + (\varepsilon_t + \alpha \varepsilon_{t-1})$$

where:

α – parameter of the autoregressive model,

β – parameter of the moving average model,

ε – model error (white noise).

Formula used in Excel:

forecast_(t) = model error_(t) + component₁ + component₂ + component₃



Model error_(t) = NORM.S.INV(rand())

component₁ = alfa * component1_(t-1) + model error_(t)

component₂ = beta * component2_(t-2) - alfa * component2_(t-1) + model error_(t)

component₃ = model error_(t) + alfa * model error_(t-1)



Autoregressive Integrated Moving Average, ARIMA

In the case of ARIMA models, attention is paid to the non-stationarity of the series. There are three parameters in the model: the autoregressive parameter (p), the moving average parameter (q) and the order of differentiation (d). The **ARIMA(p, q, d)** model is also described using numbers, for example: $(1, 1, 0)$, which means that in the series $p=1$ there is a single autoregressive parameter, $q=1$ there is a single moving average parameter and $d=0$, no differentiation occurs (Malska & Wachta, 2015). For the given example, the formal general model is as follows:

$$\tilde{y}_t = \alpha + y_{t-1} + \beta_1(y_{t-1} - y_{t-2})$$

where:

α – parameter of the autoregressive model,

β – moving average parameter.



Formula used in Excel:

forecast_(t) = alfa + demand_(t-1) + beta * [demand_(t-1) - demand_(t-2)]

Random Walk, RW

The Random Walk model is a subcategory of the ARIMA model. In a simple RW model, each forecast is assumed to be the sum of the last observation and a random error term. It therefore assumes that the latest observation is the best indication for building the nearest next forecast. This model is quite simple to understand and implement. It is used when a development trend is observed in the data sequence. This is represented by the formal model:

$$\tilde{y}_t = y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = y_{t-1} - y_{t-2}$$



where:

ε – model error (white noise).



Formula used in Excel:

$$\text{forecast}_{(t)} = \text{demand}_{(t-1)} + [\text{demand}_{(t-1)} - \text{demand}_{(t-2)}]$$

It has been observed that many complex forecasting methods based on linear structure are unable to beat the naive RW model (Adhikari & Agrawal, 2014).

Regression methods

In regression models, it is not possible to talk about the influence of one variable on another. By means of a variable or a set of variables, another variable is explained. To apply regression methods, a larger amount of historical data is needed – the longer the observation period of historical data, the greater the possibility of precisely determining forecasts.

Many variants of regression models are known: linear regression, non-linear regression, logistic regression, stepwise regression, ordinal regression. The formula for the general form of regression is:

$$\tilde{y}_t = f(X, \beta) + \varepsilon_t$$

where:

X – explanatory, predicting variable

β – regression coefficient

$f(X, \beta)$ – regression equation

ε_t – random error.

For the purposes of this study, three exponential smoothing methods will be presented: (1) Trend Projection Method; (2) Simple linear regression method; (3) Multiple linear regression method.



Trend Projection Method

The Trend Projection method is a variation of the straight line method. It is the most classic business forecasting method that deals with the movement of variables over time. A distinction can be made between the **graphical method** – in which the data is displayed on a graph and a line is drawn manually through it. The line is drawn maintaining the smallest distance between the designated points and the line; **trend equation fitting method** using data and a straight or exponential line equation.

Simple Linear Regression Method

The linear regression method is the simplest variant of regression. The purpose of the linear regression method is to fit a straight line to the data. Therefore, you need to find a solution that will allow you to find the optimal straight line that will best show the relationship between the data. In the simple linear regression method, the forecasting model is based on a linear tendency. To determine the regression line, and therefore the values in the linear regression model, you need to calculate the straight line coefficients a and b . This is represented by the formal model:

$$\tilde{y}_t = an + b$$

where:

a – value of the variable in the analysed period,

b – value of the increase or decrease in the dependent variable,

n – serial number of the analysed and forecast period.

To determine the parameters a and b , you need to calculate a system of two equations. It has the following form:

$$\begin{cases} a \sum_{i=1}^n t_i^2 + b \sum_{i=1}^n t_i = \sum_{i=1}^n t_i \cdot y_i \\ a \sum_{i=1}^n t_i + b \cdot n = \sum_{i=1}^n y_i \end{cases}$$



where:

t_i – the ordinal number of the period ($t = 1, 2, 3, \dots$), which is the value of the independent time variable,

y_i – dependent variable (e.g. demand for a given good in a given time period),

a – value of the variable in the analysed period,

b – value of the increase or decrease in the dependent variable,

n – number of all analysed periods.



However, the quickest way to calculate the above regression coefficients is to use the LINEST function.

Syntax: **LINEST(known_y,[known_x],[constant],[statistics])**

Multiple linear regression method

Multiple linear regression allows you to build models of linear relationships between many variables. The multiple linear regression method is used in data analysis to examine complex relationships between multiple variables. This is represented by the formal model:

$$\tilde{y}_t = a + b_1y_1 + b_2y_2 + \dots + b_iy_i + \varepsilon$$

where:

a – value of the variable in the analysed period,

b – value of the increase or decrease in the dependent variable,

ε – model error (white noise).

8.7. Forecast errors

To assess forecast accuracy, forecast errors must be measured. Given that there is no guarantee of perfect prediction of future demand (Hopp & Spearman, 1999), every forecast is subject to error.



Researchers distinguish between systematic and unsystematic effects of forecast errors (Zeiml, et al., 2019). The performance of a forecasting system is typically measured using various measures of forecast error (Table 8.5).

Table 8.5. Selected forecast errors

Forecast error	Model	Interpretation
Mean Squared Error (MSE)	$MSE = \frac{\sum (y_t - p_t)^2}{n}$	The mean squared error can only be positive and its value should be as small as possible. The value of this error, which is 0, indicates excellent forecast accuracy.
Root Mean Squared Error (RMSE)	$RMSE = \sqrt{\frac{\sum (p_t - y_t)^2}{n}}$	The error value should be as close to 0 as possible. The lower the root mean square error value, the better the model. And the perfect model has a value equal to 0.
Mean Absolute Percentage Error (MAPE)	$MAPE = \frac{\sum E_t - A_t }{A_t} \cdot \frac{100\%}{n}$	This is one of the more popular measures of error. A value of 0 indicates that the model has no mean error, meaning the value should be as close to 0 as possible. For the same forecast errors, smaller actual values make the relative error larger.
Mean Percentage Error (MPE)	$MPE = \frac{100\%}{n} \sum \frac{a_t - f_t}{a_t}$	This is the average percentage error (or deviation). It informs how much the deviation from the actual value will be on average during the forecast period. MPE is useful because it allows you to check whether a forecast model systematically underestimates (more negative error) or overestimates (positive error).
Mean Absolute Deviation (MAD)	$MAD = \sum \frac{ x_t - \bar{x}_t }{n}$	It is a simple extension of absolute variance. The mean absolute deviation is used as a measure of the variation in the data.

Source: (Hyndman & Koehler, 2006; Zeiml, et al., 2019)



8.8. Advantages of forecasting in Excel

Microsoft Excel can be used for forecasting. Using developed algorithms and based on collected data from the past, you can prepare forms to build forecasts and, as a result, make the right decisions in business. Excel is the basic tool for forecasting.

Table 8.6. Selected Excel Features

Function	Explanation
=AVERAGE	This function allows you to calculate an average based on existing values.
=SUM	The function allows you to calculate a sum based on existing values.
=FORECAST	The function allows you to predict future value from existing values using linear regression..
=FORECAST.ETS	Calculates or predicts future value based on existing (historical) values using a version of the exponential smoothing algorithm (ETS).
=LINEST	Calculates statistics for a line using the least squares method.
=FORECAST.LINEAR	Calculates or predicts future value from existing values using linear regression.
=TREND	It will be used to determine the linear trend.
=FORECAST.ETS.SEASONALITY	Calculates the length of the seasonal pattern based on existing values and timeline.

Source: own study

Excel is used in forecasting by many companies, small, medium and large (including corporations), because it has a number of appropriate tools at its disposal. Data can be easily stored and calculated in an Excel workbook. Excel can visualize the acquired and processed data in various ways, which is useful and helps in making forecasts easier (www_8.3).



After all, Excel uses many formulas that can be used in the programme's workbook to help calculate predicted values. Excel supports several different functions that allow you to use the software in a practical way (Table 8.6). Understanding them is key to getting the most out of Excel.

The disadvantages of Excel include the need to manually synchronize data and manually update data. Another common problem is the possibility of making mistakes as a result of incorrectly performed data import or as a result of interruption of the formula. Because Excel is a manual data entry programme, the data used for forecasting is not real-time data.

8.9. Artificial intelligence in forecasting

Supply chain data is multidimensional and generated at multiple points in the chain, for multiple purposes, in large volumes (due to the multitude of suppliers, products, and customers), and at high speed (reflecting the many transactions continuously processed across supply chain networks). This complexity and multidimensionality of supply chains is causing a shift away from conventional (statistical) approaches to demand forecasting, which are based on the identification of statistically average trends (characterized by mean and variance attributes) (Michna, i in., 2020), towards intelligent forecasts that can learn from historical data and intelligently evolve to adapt to anticipate ever-changing demand in supply chains.

The answer to new needs and challenges is anticipatory logistics, which supports processes such as demand forecasting. At its core lies the possibility of using artificial intelligence (AI). It is a combination of modern technologies such as Big Data (BD), machine learning (ML) and artificial intelligence (Sczaniecka & Smarzyńska, 2018). Technological advances in recent years have led to the increasingly frequent generation and storage of huge amounts of data. This data is captured over time at different points (e.g. at different links in the supply chain) and stored in different places. They should therefore be processed efficiently to extract useful and valuable knowledge from them (Galicia et al., 2019).



Big Data refers to dynamic, high-volume, high-velocity, and high-variety data sets that exceed the processing capabilities of traditional data management approaches (Chen et al, 2014). Big Data can provide a wealth of unique insights into things like market trends, customer purchasing patterns, and maintenance cycles, as well as ways to reduce costs and make more targeted business decisions (Wang et al, 2016). Research indicates that Big Data Analytics (BDA) can provide a way to obtain more accurate forecasts that better reflect customer needs, facilitate supply chain performance assessment, improve supply chain efficiency, shorten response times, and support supply chain risk assessment (Awwad et al, 2018).

Big Data Analytics (BDA) in Supply Chain Management (SCM) is gaining increasing attention (Seyedan & Mafakheri, 2020). Supply chain data analysis has become a complex task due to (Awwad et al, 2018):

- increasing number of SC entities,
- increasing variety of SC configurations depending on homogeneity or heterogeneity of products,
- interdependencies between these entities,
- uncertainty about the dynamic behaviour of these components,
- lack of information regarding supply chain entities,
- networked production entities due to their increasing coordination and cooperation to achieve high level of customization and adaptation to different customer needs,
- increasing use of supply chain digitalization practices (and use of Blockchain technology) to track supply chain activities.

Artificial intelligence methods are being developed very intensively around the world, both theoretically and in terms of application (Trojanowska & Malopolski, 2004). Among the artificial intelligence methods, artificial neural networks are used for forecasting. Neural networks have a number of features that are useful for analyzing and forecasting time series. Their effectiveness is primarily related to tuning the adopted structure based on data. The process of building a neural model involves exploring available data sets and enables



automatic model estimation. An additional advantage of neural networks is their ease of adaptation to changing market conditions (Cieřak et al, (2006).

Unlike traditional model-based methods, **artificial neural networks** (Artificial Neural Networks, ANNs) are self-adaptive data-driven methods. They learn from examples and capture subtle functional relationships between data, even if the underlying relationships are unknown or difficult to describe. Artificial neural networks can generalize (Hornik et al., 1989). After examining the presented data, they can infer the unseen part of the population, even if the sample data contains noisy information. They also have more general and flexible functional forms than traditional statistical methods (Zhang et al., 1998).

Chapter Questions

1. What are the limitations of using time series methods in demand forecasting?
2. What are the main components of a time series and what is their importance in the forecasting process?
3. What are the different strategies for dealing with outlier data?

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